

# Principal Component Analysis for Spectral Indices of Stellar Populations

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Accepted . Received.

## ABSTRACT

We apply the method of principal component analysis to a sample of simple stellar population to select some age sensitive spectral indices. Besides the well-known age sensitive index,  $H\beta$ , we find some other spectral indices have great potential to determine the age of stellar population, such as G4300, Fe4383, C<sub>2</sub>4668, and Mgb. In addition, we find these spectral indices sensitivity to age depends on the metallicity of SSP,  $H\beta$  and G4300 are more suited to determine the age of low metallicity stellar population, C<sub>2</sub>4668 and Mgb are more suited to the high metallicity stellar population. The results suggest that principal component analysis method provides a more objective and informative alternative to diagnostics by individual spectral lines.

### Key words:

methods: data analysis – galaxies: evolution – galaxies: fundamental parameters – galaxies: star clusters.

## 1 INTRODUCTION

To understand how galaxies are formed and evolve we need to investigate their stellar populations and to derive their main parameters, such as metallicity and age. This study plays an important role in our understanding of galaxy properties (Kennicutt 1998; Maraston 1998; Padoan et al. 1997). During the last decade particularly using population synthesis method has performed determination of galaxy properties. It has been used extensively by many authors for all kinds of galaxies (Leitherer et al. 1996). However, key parameters in interpretation of the observed properties are the metallicity and the age. The problem is the degeneracy of the effects from variations in age and in metallicity, even in the simplest unresolved stellar systems their effects are very difficult to separate (Vazdekis et al. 1997). It makes the determination of age and metallicity uncertain. To determine the age and metallicity more accurately, strong efforts have been devoted to select some spectral features that are more sensitive to age, and others are more sensitive to metallicity. In 1994, G. Worthey developed a very efficient method to investigate the age and metallicity effects in the integrated light of stellar populations. In his method, suppose an index varies by an amount  $\Delta I$ , it can be explain  $\Delta I$  by either a pure metallicity variation or a pure age variation. The Z sensitivity parameter is the ratio of the percentage change in Z, with larger numbers indi-

cating greater metallicity sensitivity (Worthey 1994). Using this method, some metallicity sensitive spectral indices are found, but only two age sensitive spectral indices are found.

In this paper, we apply a different statistical technique, principal component analysis (PCA) to simple stellar population sample, to investigate on the reliability of the relations between spectral indices and age. We want to extract other spectral indices that can be used to determine the age of stellar population. The samples of simple stellar population (SSP) spectra come from Bressan et al. (1996) and Bruzual & Charlot (1996). For each spectra, 21 spectral indices in the Lick/IDS system be measured first (Trager et al. 1998), then we use PCA method to find some spectral indices that are more sensitive to the age. We leave the synthetic galaxy spectral indices with different star formation history and observational galaxy spectral indices to a future paper. However we already notice an encouraging good resemblance between the results of PCA and those one gets from observation (Worthey 1994).

The organization of the rest of this paper is as follows. In section 2 we present PCA method. In section 3 we describe the definitions of spectral indices, the synthetic models we have used. In section 4 we analyze a sample of SSP spectra with different ages, find the first principal components (PCs), discuss the significance of different spectral indices to determine the age, and how different age sampling affects the results. In section 5, we illustrate that the uncertainties

will not affect our results, and compare our results with previously published values. The conclusions are summarized in section 6.

## 2 PRINCIPAL COMPONENT ANALYSIS

Principal component analysis is known to be powerful technique for unveiling correlation between variables in a data set and for determining the intrinsic dimensionality of a parameter space (Connolly & Szalay 1999). It is an orthogonal transformation that allows the building of more compact linear combinations of data that are optimal with respect to the mean square error criterion. The algorithm calculates a new base with the minimum set of orthogonal axes that describes the observed variance of the data. Finding the new axes is an eigenvalue formation matrix between the old base and the new base. The new orthogonal basis is composed of vectors called principal components (PCs). The corresponding eigenvalues represent the variances of the parameters in the new bases (Francis & Wills 1999).

The formulation of standard PCA is as follows (see e.g. Murtagh & Heck 1987). Consider a set of  $n$  objects ( $i = 1, n$ ), each with  $m$  parameters ( $j = 1, m$ ). If  $x_{ij}$  are the original measurements, the starting point in PCA method is an observation matrix  $X = (x_{ij})_{n \times m}$  in which row vectors list observations characterizing an object.

We then construct a covariance matrix  $C = (c_{ij})_{m \times m}$ , where

$$c_{jk} = \frac{1}{n-1} \sum_{i=1}^n (x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k) \quad 1 \leq j, k \leq m \quad (1)$$

where  $\bar{x}_j$  and  $\bar{x}_k$  are the mean of column vector.

It can be shown that the axis along which the variance is maximal is the eigenvector  $e_1$  of the matrix equation

$$Ce_1 = l_1 e_1, \quad (2)$$

where  $l_1$  is the largest eigenvalue of matrix  $C$ , which is in fact the variance along the new axis. The other principal axes and eigenvectors obey similar equations. It is convenient to sort them in decreasing order, and to quantify the fractional variance by  $l_j / \sum_j l_j$ . The matrix of all the eigenvectors forms a new set of orthogonal axes that are ideally suitable to a description of the data set. If the main variance of the data set lies in a small dimensional space, then one can get a good visualization of it by plotting the data projected on the first few eigenvectors (PCs). The projection of a vector  $x$  on the eigenvector  $e_j$  being  $x \cdot e_j$ , where  $x$  is a row vector of  $X$ . The fractional variance of the first PCs tells us to what extent the data lie in a given low dimensional space (Ronen et al. 1999).

In what follows, we will apply this technique to different spectral index samples. To avoid confusion we denote eigenvectors by PC1, PC2 etc., and projections on the new axes by pc1, pc2 etc. For the  $k$ th object,  $pc_j = x \cdot e_j = e_{j1}x_{k1} + \dots + e_{jm}x_{km}$ .

## 3 INDICES OF SIMPLE STELLAR POPULATION

Among the libraries of spectral indices, the most extensive one is created by Faber and coworkers at Lick Observatory, the Lick/IDS System (Faber et al. 1985; Worthey et al. 1994; Trager et al. 1998). In the Lick/IDS system, absorption-line strengths are described by “indices,” in which a central “feature” band-pass is flanked to the blue and red by “pseudo-continuum” band-passes. It is fully described in Worthey et al. (1994) and Trager et al. (1998). Table 2<sup>\*</sup> of Trager et al. (1998) presents the band-passes of these 21 Lick/IDS absorption line indices and the features measured by these indices.

Simple stellar population is defined as a single generation of coeval stars with fixed parameters such as metallicity, initial mass function, etc (Buzzoni 1997). To find some spectral indices that can be used to determine the age of stellar population, as the first step, we apply PCA method to SSP only. To assess how model-dependent our conclusions are, we use the spectral indices of SSP form two different synthetic models.

The first SSP sample is the one used by Tantalo et al. (1996). Detailed descriptions of the SSP model can be found in Bressan et al. (1994), Silva (1995), and Tantalo et al. (1996). These SSPs extend the ZAMS to  $0.15 M_\odot$ . The rate of mass loss during the RGB and TP-AGB phases is according to the Reimers (1975) relationship with  $\eta = 0.45$ . They are computed with the new library of stellar spectra described in Silva (1995), and Tantalo et al. (1996). These SSPs are shortly referred to as SIL-SSP. The spectral indices of this sample are calculated by means of the empirical calibrations of Worthey (1992) and Worthey et al. (1994).

The second one is isochronal synthesis model of Bruzual & Charlot (1996). It has extended the Bruzual & Charlot (1993) evolutionary population synthesis model. The updated version provides the evolution of the spectrophotometric properties for a wide range of stellar metallicity. The stellar evolution tracks are from Padova group (Bressan et al. 1993). The stellar spectra library is from Lejeune et al. (1997, 1998). These SSPs are shortly referred to as BC-SSP.

The spectral indices in Lick/IDS system for SIL-SSP and BC-SSP are published in the AAS CD-ROM Series, Vol. 7. The technique to calculate spectral indices of SSP is amply described in Worthey (1994), Bressan et al. (1996), Tantalo et al. (1998) and Longhetti et al. (1998).

## 4 PCA TO SSP

To find some spectral indices that are more sensitive to the age, we will apply PCA method to SSPs in this section.

### 4.1 PCA to SIL-SSP

Given an IMF and metallicity, evolutionary synthesis model provides SSP’s spectra as a function of age in the sample of SIL-SSP. Each data set contains about 50 SSPs, corresponding to 50 time steps from 0.004 to 20 Gyr. Each SSP

<sup>\*</sup> This table is available electronically from the Astrophysical Data Center (<http://adc.gsfc.nasa.gov/adc.html>).

has 1206 wavelength points in the range from 91 Å to 100  $\mu\text{m}$  (Leitherer et al. 1996). Because the spectral indices of CN<sub>1</sub>, CN<sub>2</sub> and TiO<sub>2</sub> are equal to 0.0 or very little for young stellar population, and there are some uncertainties at high  $Z$  SSP, so we perform PCA on a sample of SSP with age 1–15 Gyr, and  $Z=0.0004$  to 0.05.

For our first study, we performed PCA on the SIL-SSP with  $Z = Z_{\odot} = 0.02$ . The results are shown in Table 1. Column (2)–(3) show the first 2 principal components (PCs) out of a total of 21 principal components. The 3rd row in Table 1 gives the variances (eigenvalues) of the data along the direction of the corresponding principal component. By convention, the principal components are given in order of their contribution to the total variance, it is given as ‘Proportion’ in the 4th row. The columns of numbers for each principal component represent the weights assigned to each input variable. Thus  $\text{pc1} = 0.02 \times x_1 + 0.01 \times x_2 + 0.14 \times x_3 + 0.49 \times x_4 + 0.49 \times x_5 \dots$ , where  $x_1, x_2, x_3, x_4$ , and  $x_5 \dots$ , are the values of the normalized variables corresponding to CN<sub>1</sub>, CN<sub>2</sub>, Ca4227, G4300 and Fe4383, etc. If the value in Column 2 of Table 1 is large, it suggests that the corresponding spectral index is important to PC1. The first principal component is elongated with variance 5.11, and accounts for about most the total variance. It is therefore likely to be highly significant. In SIL-SSP sample, only age is variable, so PC1 shows the age information, it correlates with ages. We can use PC1 to determine the age of a SSP.

This result shows that most of these spectral indices correlate with PC1, but the correlation involving G4300, Fe4383, C<sub>2</sub>4668, H $\beta$ , and Mgb are very strong. So these spectral indices can be used to determine the age of stellar system. PC2 accounts for only 1% of the variance in this data. Some spectral indices appear to contribute to both PC1 and PC2, but the contribution of PC2 is very little, therefore any spectral indices present in PC2 represent only fine tuning to the main variation by which PC1 represents. PC2 is not a significant component.

To inspect how sensitive the results of PCA method are to the different metallicity, we performed PCA on the SIL-SSP with other metallicity. The results of performing PCA on these SSPs are shown in the last 8 columns of Table 1. There are some distinct characters in the results. First, we find that the spectral indices G4300, Fe4383, C<sub>2</sub>4668, H $\beta$ , and Mgb are very strong correlated with PC1 also. Second, the contribution from PC1 is much more important than that from PC2. Third, we find that the spectral indices contributing to the PCs are different from the SSPs with different metallicity. Mgb and C<sub>2</sub>4668 contributing to low metallicity SSP are small, they cannot fit to determine the age for low metallicity SSP but for high metallicity SSP. On the contrary, G4300 and H $\beta$  are more fit to determine the age for low metallicity SSP. Actually, the sodium index NaD is also an important index, with respect to the PC1. We did not select it due to the considerable effect of the interstellar sodium line as previously reported (Worley 1994).

## 4.2 PCA to BC-SSP

There exist a number of SSP models that are synthesized with different approaches. It is important to check the sensitivity of our results to the SSP models adopted. The Bruzual & Charlot’s model is suitable for the comparison, because it

uses a similar technique of “isochronal synthesis” to predict the spectral evolution of stellar population. Each data set (metallicity fix) contains about 221 SSPs, corresponding to 221 time steps from 0 to 20 Gyr. Each SSP has 1206 wavelength points in the range from 5 Å to 100  $\mu\text{m}$  (Bruzual & Charlot 1996). Similar to SIL-SSP, we performed PCA method to the stellar population (1–15 Gyr) with metallicity  $Z=0.0004$  to 0.05. The results are shown in Table 2.

In Table 2, the first principal component also accounts for about most the total variance, more than 98%. PC2 can be ignored, it maybe caused some uncertainties. This result is consistent with the principle of PCA method: One of important application of PCA is known to be powerful technique for determining the intrinsic dimensionality of a parameter space. Given an IMF and metallicity, SSP has one parameter, age. This parameter can be expressed by PC1. From Table 1 and Table 2, we find the results from different SSP are very similar, it can be explained that our results are not sensitive to the SSP models adopted. The spectral indices, G4300, Fe4383, C<sub>2</sub>4668, H $\beta$ , and Mgb, can be used to determine the age of stellar population.

## 4.3 PCA to the SSPs with different metallicity

To extract an objective information on the age-metallicity sensitivity of the narrow band indices, we apply PCA method to the whole set of SSPs. As a result, we find a little information for metallicity. We think it can be understood easily, because there are only five kind metallicities for the sample of SSP, the metallicity point is less than the kind of spectral indices. So we can not obtain more metallicity information from this statistical technique. In the future paper, we will observe some spectra of globular cluster with different metallicity, and perform a principal component analysis method to them. It maybe help us to get the metallicity sensitive indices.

To overcome this difficulty, we have selected 3 spectral indices from Worley (1994). There are H $\beta$  which is age sensitive, Fe5015 which is metallicity sensitive and Ca4227 which isn’t age or metallicity sensitive. We apply PCA method to these 3 indices, with same age and different metallicities (5 kind metallicities). The results are shown in Column (3)–(8) of Table 3 for SIL-SSPs, and in Column (9)–(14) for BC-SSPs. The age of SSP is  $T = 10^9$  yr,  $10^{10}$  yr, and  $1.5 \times 10^9$  yr. The 1st row in Table 3 gives the age of the SSP, the meaning of next 3 rows same as Table 1. The numbers in the last 3 rows represent the weights assigned to each input variable. If the value in these columns is large, it suggests that the corresponding spectral index is important. From this table, we find the first principal component accounts for about most the total variance. This is therefore to be highly significant. In this SSP sample, since the age of SSP is same, only metallicity is variable, so PC1 must show the metallicity information. In the table, the value of the normalized variable corresponding to Fe5015 is large, it suggests that the corresponding spectral index, Fe5015, is important to metallicity. The value of H $\beta$  is small, it seems that H $\beta$  is not important to metallicity. This result is the same as the result of Worley (1994). From this analysis, we know PCA is a very efficient method, it can be used to select some spectral indices that will help us to determine the age

**Table 1.** Results of Eigenanalysis for SIL-SSP (Padova group): Percentage of variance explained by the principal components and Weights on the first two components for each index involved in the analysis. See Trager (1998) for index definition.

Metallicity	Z=0.02		Z=0.0004		Z=0.004		Z=0.008		Z=0.05	
Index	PC1	PC2	PC1	PC2	PC1	PC2	PC1	PC2	PC1	PC2
Eigenvalue	5.11	0.02	2.86	0.02	5.25	0.03	5.09	0.02	5.06	0.04
Proportion	99.42%	0.47%	99.01%	0.63%	99.35%	0.51%	99.39%	0.40%	99.04%	0.78%
Variable	PC1	PC2	PC1	PC2	PC1	PC2	PC1	PC2	PC1	PC2
01 CN <sub>1</sub>	0.02	0.02	0.02	-0.02	0.01	0.01	0.01	0.01	0.02	0.04
02 CN <sub>2</sub>	0.01	0.02	0.01	-0.01	0.01	0.01	0.01	0.02	0.02	0.05
03 Ca4227	0.14	0.25	0.06	0.13	0.08	-0.21	0.10	-0.22	0.18	0.26
04 G4300	0.49	-0.53	0.79	0.05	0.67	0.18	0.60	-0.01	0.35	-0.70
05 Fe4383	0.49	0.39	0.33	0.16	0.44	-0.29	0.46	-0.34	0.50	0.22
06 Ca4455	0.12	0.02	0.04	0.04	0.10	-0.03	0.11	0.00	0.13	0.01
07 Fe4531	0.17	0.05	0.22	-0.16	0.19	0.12	0.18	0.13	0.17	0.07
08 C <sub>2</sub> 4668	0.36	-0.10	-0.06	0.37	0.22	0.04	0.29	0.36	0.42	-0.11
09 H $\beta$	-0.32	0.33	-0.41	0.22	-0.39	-0.08	-0.37	-0.16	-0.27	0.25
10 Fe5015	0.19	-0.27	0.16	-0.16	0.19	0.50	0.20	0.55	0.19	-0.25
11 Mg1	0.01	0.02	0.00	0.02	0.01	-0.02	0.01	-0.01	0.01	0.02
12 Mg2	0.02	0.02	0.00	0.03	0.01	-0.03	0.01	-0.02	0.02	0.02
13 Mgb	0.28	0.31	0.07	0.65	0.16	-0.33	0.22	-0.29	0.34	0.22
14 Fe5270	0.15	0.07	0.08	0.03	0.13	0.01	0.14	0.09	0.15	0.07
15 Fe5335	0.14	0.06	0.09	0.08	0.12	-0.15	0.14	0.06	0.15	0.09
16 Fe5406	0.11	0.06	0.03	0.09	0.08	0.00	0.09	0.08	0.12	0.03
17 Fe5709	0.04	-0.01	0.01	-0.01	0.04	0.06	0.04	0.09	0.03	-0.01
18 Fe5782	0.03	-0.01	0.00	0.03	0.03	0.00	0.03	0.10	0.04	0.01
19 NaD	0.22	0.45	0.03	0.53	0.11	-0.65	0.15	-0.49	0.29	0.43
20 TiO <sub>1</sub>	0.00	0.00	0.00	0.01	0.00	-0.01	0.00	0.00	0.00	0.00
21 TiO <sub>2</sub>	0.00	-0.01	0.00	0.02	0.00	-0.01	0.00	0.01	0.01	-0.02

**Table 2.** Results of Eigenanalysis for BC-SSP (Bruzual & Charlot 1996)

Metallicity	Z=0.0004		Z=0.004		Z=0.008		Z=0.02		Z=0.05	
Index	PC1	PC2	PC1	PC2	PC1	PC2	PC1	PC2	PC1	PC2
Eigenvalue	1.17	0.02	2.68	0.04	3.39	0.02	4.66	0.02	5.06	0.06
Proportion	98.25%	1.37%	98.30%	1.56%	99.16%	0.68%	99.54%	0.37%	98.73%	1.24%
Variable	PC1	PC2	PC1	PC2	PC1	PC2	PC1	PC2	PC1	PC2
01 CN <sub>1</sub>	0.00	0.00	0.00	0.00	0.00	-0.01	0.01	-0.02	0.01	0.05
02 CN <sub>2</sub>	0.00	0.00	0.00	-0.01	0.00	-0.01	0.01	-0.03	0.02	0.06
03 Ca4227	0.10	0.17	0.12	-0.03	0.12	0.02	0.13	-0.28	0.17	0.20
04 G4300	0.34	0.17	0.57	0.33	0.54	0.34	0.48	0.68	0.32	-0.81
05 Fe4383	0.18	0.15	0.39	0.37	0.46	0.40	0.51	-0.18	0.47	-0.03
06 Ca4455	-0.07	0.12	0.13	-0.04	0.12	-0.09	0.11	-0.09	0.11	0.01
07 Fe4531	-0.23	0.26	0.22	-0.23	0.19	-0.25	0.18	-0.12	0.18	0.07
08 C <sub>2</sub> 4668	-0.01	0.27	0.18	0.12	0.29	-0.01	0.38	-0.07	0.51	0.24
09 H $\beta$	-0.83	0.45	-0.42	0.70	-0.32	0.64	-0.26	-0.16	-0.22	0.14
10 Fe5015	-0.21	0.41	0.24	-0.29	0.24	-0.36	0.21	0.10	0.21	-0.13
11 Mg1	0.00	0.01	0.01	0.00	0.01	0.00	0.01	-0.03	0.01	0.01
12 Mg2	0.01	0.03	0.02	0.01	0.02	0.00	0.02	-0.04	0.02	0.02
13 Mgb	0.11	0.44	0.28	0.28	0.31	0.18	0.31	-0.11	0.32	0.04
14 Fe5270	0.11	0.12	0.17	-0.10	0.16	-0.17	0.15	-0.15	0.15	0.10
15 Fe5335	0.11	0.24	0.16	-0.03	0.16	-0.17	0.15	-0.14	0.16	0.06
16 Fe5406	0.05	0.07	0.11	-0.02	0.10	-0.11	0.11	-0.14	0.12	0.04
17 Fe5709	0.03	-0.01	0.03	-0.07	0.03	-0.05	0.04	-0.03	0.03	-0.01
18 Fe5782	0.02	-0.01	0.04	-0.05	0.03	-0.09	0.03	-0.05	0.04	0.04
19 NaD	0.10	0.35	0.16	0.13	0.18	-0.03	0.20	-0.52	0.28	0.42
20 TiO <sub>1</sub>	0.00	0.01	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00
21 TiO <sub>2</sub>	0.00	0.02	0.01	0.00	0.01	-0.02	0.00	0.00	0.01	0.00

**Table 3.** Results of Eigenanalysis for the SSP with different metallicity.

Age	SIL-SSP						BC-SSP					
	10 <sup>9</sup> yr		10 <sup>10</sup> yr		1.5 × 10 <sup>9</sup> yr		10 <sup>9</sup> yr		10 <sup>10</sup> yr		1.5 × 10 <sup>9</sup> yr	
Index	PC1	PC2	PC1	PC2	PC1	PC2	PC1	PC2	PC1	PC2	PC1	PC2
Eigenvalue	2.96	0.03	3.87	0.02	4.53	0.06	3.35	0.02	4.44	0.01	4.69	0.01
Proportion	98.8%	1.16%	99.5	0.52	98.8	1.20	99.3	0.66	99.8	0.13	99.7	0.27
Variable	PC1	PC2	PC1	PC2	PC1	PC2	PC1	PC2	PC1	PC2	PC1	PC2
03 Ca4227	0.17	-0.15	0.34	0.91	0.32	0.56	0.16	0.21	0.28	0.94	0.31	0.95
09 H $\beta$	-0.32	0.93	-0.21	0.32	-0.30	0.83	-0.50	0.86	-0.19	0.24	-0.17	0.05
10 Fe5015	0.93	0.35	0.92	-0.26	0.90	0.07	0.85	0.46	0.94	-0.23	0.93	-0.31

or metallicity of SSP, if the age and metallicity points are enough.

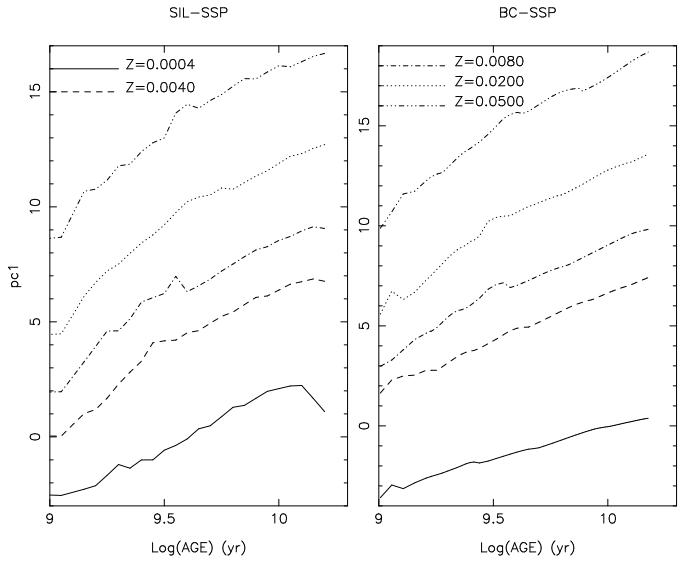
#### 4.4 PC1 and Index with Age

For each SSP sample (metallicity fix), we can obtain  $\vec{e}_j$  from PCA method. Using  $pc_j = \vec{x} \cdot \vec{e}_j$ , we can calculate the projection of a vector  $\vec{x}$  on the eigenvector. We plot the projection on the PC1 (pc1) versus age for the metallicity range from Z=0.0004 to Z=0.05 in Fig. 1. Each line pattern represents a different metallicity, as indicated inside the frame. In Fig. 1(a), we plot the pc1 versus age for SIL-SSP. In Fig. 1(b), we plot for BC-SSP. In these figures, the following remarks can be made. (1) It is apparent that there is a uniform tendency for the values of pc1 to become larger as the metallicity increases from Z = 0.0004 to Z = 0.05. (2) These mainly are monotonically increasing curve with age, clearly. Therefore, once we know the metallicity, we can use this pc1 to determine the age of stellar population. (3) Some small non-monotonic irregularities in the curve are probably due to abrupt transition of stars to different evolutionary stages. They may be real, but they may also occur if the model does not contain enough evolutionary tracks in a certain range. In any case, we shall see that these irregularities will be smoothed out when considering longer and more realistic star formation histories instead of the instantaneous bursts considered here.

As an example, we plot the spectral indices with age for SIL-SSP with Z=0.02 in figure 2. We find that spectral indices become stronger with increasing age, while Balmer line, H $\beta$ , become weaker. The change of G4300, Fe4383, C<sub>2</sub>4668, H $\beta$ , and Mg $\beta$  with age are very strong, especially for G4300 and H $\beta$ . These results, from Fig. 1 and Fig. 2, consistent with the results from PCA method. So these spectral indices can be used to determine the age of stellar system. The results suggested that PCA is efficacious method for find the age sensitive indices.

## 5 DISCUSSION

In this paper, as the first step, we only performed PCA to SSP sample. There are two reasons. Firstly, SSP is simple and reasonably well understood, so it is important to see what one can learn by using this simplest assumption, and then check whether more complex star formation histories

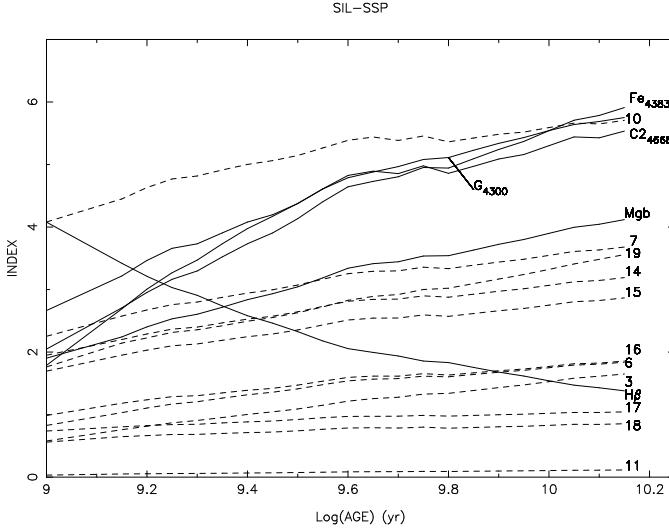


**Figure 1.** Projection of the spectral indices on the PC1 (pc1) versus age. The synthetic spectral indices of SSP formed in a single instantaneous burst. a) SIL-SSP, b) BC-SSP.

give qualitatively similar conclusions. This is a common approach often taken in the evolutionary population synthesis models for galaxies (Vazdekis et al. 1997). Secondly, the selected spectral indices in this paper can be used to determine the age of star cluster.

The synthesis spectral indices are computed from the SSP of various ages and metallicity, under some fitting functions. The uncertainties in the fitting function, the stellar evolution input and the evolutionary population synthesis computational procedure may be affecting our result. But for the result of Maraston et al. (1999), the discrepancies introduced by different fitting function are relatively small. Form the result state above , the difference between the SIL-SSP and BC-SSP is very small, so the uncertainties due to the use of different evolutionary population synthesis procedures and stellar evolution input can be ignored.

Compared with the previous methods, PCA method provides a more objective and informative alternative to diagnostics by individual spectral lines, the PCs represent all the spectral features *simultaneously*. Same as Worthey (1994), we also find G4300 and H $\beta$  are age sensitive indices that are useful in determining age. In addition, we also find



**Figure 2.** Spectral indices of the SIL-SSP versus age. The solid lines show the spectral indices that can be used to determine the age of stellar population. We use the spectral index name or serial number (in Tab. 1) to indicate it. Some index is not plotted in this figure for its change little.

some other indices that can be used to determine the age of SSP with different metallicities.

## 6 CONCLUSIONS

In order to select spectral indices that more sensitive to age than metallicity, PCA method has been applied to the Lick indices in the population model of SIL-SSP and BC-SSP. It is the first time this method is used for selecting the age sensitive spectral indices. Below, we summarize our main conclusions.

(i) Using PCA method, we find some spectral indices, such as G4300, H $\beta$ , C24668 and Mg $B$ , which can be used to determine the age of stellar population (1–15 Gyr). These spectral indices will help us to determine the age of stellar population more accurate.

(ii) Important differences are found between the age sensitive indices at different metallicities. For example, in low metallicity C24668 and Mg $B$  are not fit to determine the age of stellar population, but can be used to determine the age for high metallicity stellar population.

(iii) To understand how sensitive our method is to different stellar population synthesis models and metallicities, we have compared the results obtained with SIL-SSP and BC-SSP with 5 kind of metallicities. We find that although the precise values of PCs are difference, the general trend is very similar. We can conclude that the uncertainties in the evolutionary population synthesis models and metallicities can be ignored.

## ACKNOWLEDGMENTS

We deeply thank the referee, Dr. A. Bressan, for so carefully reading an early draft of this paper and for making some very useful comments and constructive suggestions,

which improved the manuscript. We are indebted to Dr. S. Ronen and Dr. G. Worthey to have made available their program. We are grateful to Dr. A. Bressan for providing us with a grid of spectral indices for simple stellar population. This work is supported by the Chinese National Natural Science Foundation (CNSF) and Chinese National Pandeng Project.

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